Name

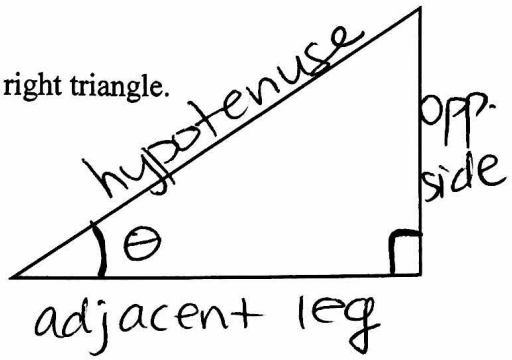
The Tangent Ratio

Lesson Objective: IWBAT apply the tangent ratio in right Δs to find side lengths.

A trigonometric ratio is a ratio of two sides of a right triangle.

The sides that we compare are all in relation to an acute angle in the right triangle.

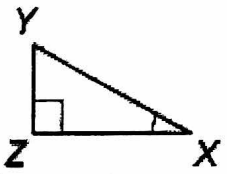
These sides are called opp. leg, adjacent leg, and hypotenuse.



There are ways in which we can compare combination of these two sides in a triangle, and each of these ratios have a specific name.

The Tangent Ratio

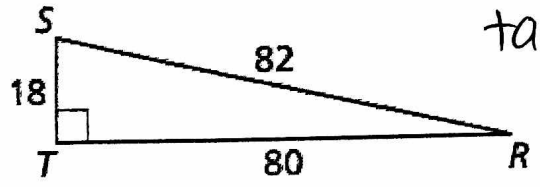
The tangent ratio of an acute angle compares the legs of the right triangle in the following way:



$$\tan\theta = \frac{\text{opp. leg}}{\text{adj. leg}} \qquad \tan X = \frac{YZ}{XZ}$$

Example:

1. Find tan *S* and tan *R*. Leave each answer as a fraction simplified completely.



$$\tan S = \frac{80}{18} = \frac{40}{9}$$

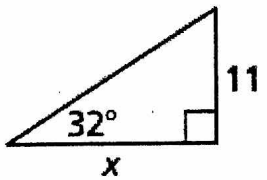
$$\tan R = \frac{18}{80} = \frac{9}{40}$$

All right triangles with a given acute angle are similar by AA~. So, regardless of the size of the triangle, the tangent ratios will ALWAYS be the same for any given acute angle measure.

We can use this fact to find missing side lengths in right triangles.

Examples:

2. Find the value of x. Round your answer to the nearest hundredth.

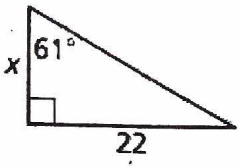


$$\frac{\tan(32)}{1} = \frac{11}{x}$$

$$x(\tan(32)) = 11$$

$$x = \frac{11}{\tan(32)} \approx 17.60$$

3. Find the value of x. Round your answer to the nearest hundredth.

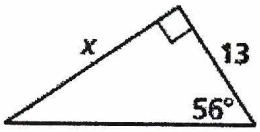


$\tan(61) = \frac{22}{x}$

$x(\tan(61)) = 22$

$x = \frac{22}{\tan(61)}$ ≈ 12.19

4. Find the value of x. Round your answer to the nearest hundredth.



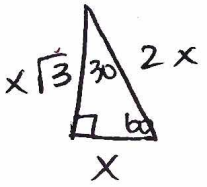
$\tan(56) = \frac{x}{13}$

$x = 13(\tan(56))$ ≈ 19.27

Tangent Ratios in Special Right Triangles

Example:

5. Use a special right triangle to find tan(60°).



$\tan(60) = \frac{x\sqrt{3}}{x}$

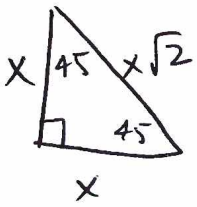
$\tan(60) = \sqrt{3}$

6. Use a special right triangle to find tan(30°).

$\tan(30) = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

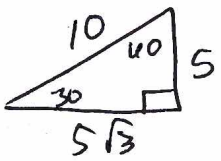
$\tan(30) = \frac{\sqrt{3}}{3}$

7. Use a special right triangle to find tan(45°).



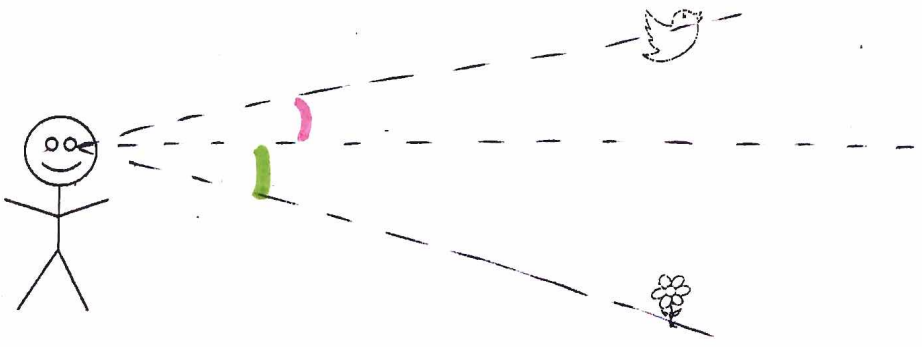
$\tan(45) = \frac{x}{x}$

$\tan(45) = 1$



Angle of Elevation: The angle that an upward line of sight makes with a horizontal line

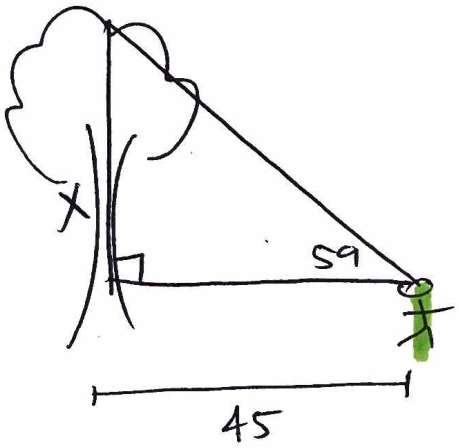
Angle of Depression: The angle that a downward line of sight makes with a horizontal line



ıg Tangent Ratios with Math Modeling

Example:

8. You are measuring the height of a spruce tree. You stand 45 feet from the base of the tree. Your view to the top of the tree has an angle of elevation of 59°. Find the height of the tree to the nearest foot. Draw a sketch of the scenario.



$$\tan(59) = \frac{x}{45}$$

$$x = 45(\tan(59))$$

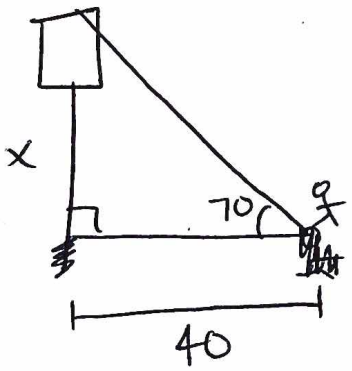
$$x \approx 74.89 \text{ ft}$$

$$+5$$

$$\approx 79.89 \text{ ft} \rightarrow \boxed{80 \text{ ft}}$$

*consider eye height! (5 ft above ground)

9. You are measuring the height of a lamppost. You stand 40 inches from the base of the lamppost. You measure the angle of elevation from the ground to the top of the lamppost to be 70°. Find the height of the lamppost to the nearest inch. Draw a sketch of the scenario.

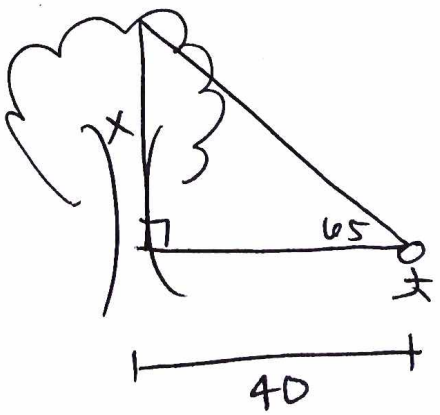


$$\tan(70) = \frac{x}{40}$$

$$x = 40(\tan(70))$$

$$x \approx 109.90 \rightarrow \boxed{110 \text{ in}}$$

10. You are measuring the height of a tree. You stand 40 feet from the base of the tree. The angle of elevation to the top of the tree is 65°. Find the height of the tree to the nearest foot. Draw a sketch of the scenario.



$$\tan(65) = \frac{x}{40}$$

$$x = 40(\tan(65))$$

$$x \approx 85.78$$

$$+5$$

$$90.78 \rightarrow \boxed{91 \text{ ft}}$$